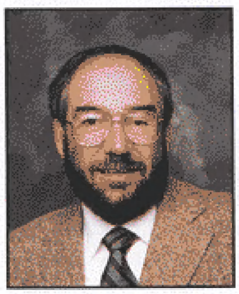
The Synchronous Amplification Factor, Split Resonances, and Virtual Probe Rotation



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Charlie obtained a B.S. and an M.S. in Mechanical Engineering from the University of California at Berkeley. During his undergraduate education, he worked as an aerospace mechanism designer at Lockheed Missiles and Space Company. His graduate research involved modeling and simulation of an automotive hydraulic valve lifter with oil compressibility effects. After joining Bently Nevada Corporation in 1989, Charlie has worked as an Engineer and Transducer Team Leader in Custom Products, and as a Research Engineer and Laboratory Supervisor in Bently Rotor Dynamics Research Corporation (BRDRC). At BRDRC, Charlie conducted research in the Direct Stiffness of fluid-lubricated bearings, applications of the Root Locus to rotor instability, the influence of fluid-lubricated bearings on polar plot balancing, the behavior of rotor systems with anisotropic properties, and the influence of rotor system gyroscopics on fluid instability. Charlie now works as an Instructor/Developer with the Bently Nevada Machinery Diagnostics Training Department.

The Synchronous Amplification Factor (SAF) is a commonly used measure of how a rotating machine behaves when passing through a balance resonance. It is usually measured using simple, graphical methods on a 1X-filtered Bode or polar plot. However, the observed SAF can become difficult to interpret when split resonances exist in a machine. Also, recent research [1,2] by the Bently Rotor Dynamics Research Corporation (BRDRC) has demonstrated that, when split resonances exist, the measured SAF can become highly dependent on the angular mounting orientation of the observing vibration transducers. Split resonances can also make polar plot balancing more difficult because low speed response vectors may not point toward the heavy spot.

This article discusses a new method that can be used under these conditions to create a set of "virtual" probes at any mounting orientation. These virtual probes can help in evaluating the SAF and also help resolve the polar plot balancing problem.

What is the Synchronous Amplification Factor?

The Synchronous Amplification Factor is measured by the ratio of the vibration amplitude at a balance resonance to the vibration amplitude at speeds well above the resonance. The word "Amplification" refers to the fact that vibration is amplified when a vibrating system passes through a resonance. The term "Synchronous" refers to the fact that the vibration is a result of a rotating force (unbalance) which rotates at the same speed as (is synchronized with) the rotor. A machine with a large SAF will have a sharp, high amplitude balance resonance peak associated with a rapid change in phase. For example, a machine with an SAF of 10 would have a peak amplitude of vibration ten times the vibration level at high speed, well above the resonance. Conversely, a machine with a small SAF will have a relatively low maximum vibration amplitude at resonance, accompanied by a relatively slow change in phase.

The SAF is often considered to be a measure of the damping of a machine, but, actually it is more complicated than that. The maximum amplitude of a rotor system at a balance resonance is controlled by the Quadrature Dynamic Stiffness of the machine. The Quadrature Dynamic Stiffness is a function of both the damping, D, and the Fluid Circumferential Average Velocity Ratio, λ. In machines with a significant amount of fluid circulation, either as process fluid or in fluid-lubricated bearings or seals, the effective damping of the system is almost always significantly smaller than the value of D alone would indicate. Thus, it is more correct to say that the SAF is a measure of the Quadrature Dynamic Stiffness of the rotor system.

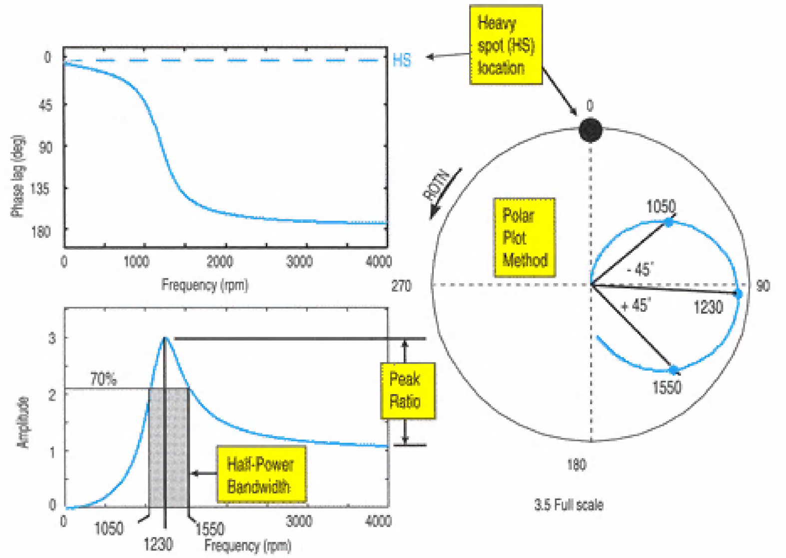

Figure 1

1X-filtered Bode and polar plot of the response of an ideal, isotropic rotor model. The response is as viewed by a vertical probe, and the heavy spot is located at 0° (under the vertical probe) when the Keyphasor pulse occurs. The polar plot from a horizontal probe would appear identical. Three graphical methods for calculating the SAF are shown. The details of the methods are explained in the text.

The SAF has become important in machine acceptance testing. The American Petroleum Institute (API) sets limits on the allowable maximum SAF for various machines. When these limits are incorporated into legal contracts between manufacturers and customers, the measurement of the SAF can become important in satisfying contractual requirements.

There are practical uses for the SAF as well. Because the dominant vibration in most healthy machines is due to unbalance, a high SAF means that a relatively small unbalance can produce large vibration amplitudes in the vicinity of a resonance. Thus, a machine with a high SAF might be more sensitive in the resonance region to changes in balance state for that mode. Also, machines which operate above a balance resonance need to be able to get through the resonance without damage. A machine with a high SAF could be vulnerable to rubbing while passing through a resonance during startup or shutdown. Thus, the SAF can be used as a rough guide for setting alarm limits when running above a resonance.

In the past, the Synchronous Amplification Factor has been considered a measure of the inherent stability of a machine. This is not true. For example, a machine with rolling element bearings might have low Quadrature Dynamic Stiffness and a high SAF. But, if there is no fluid interaction in the machine, the machine would be immune to fluid-induced instability. On the other hand, there are many examples of machines with low SAFs which are plagued with fluid instability problems.

Measurement of the Synchronous Amplification Factor

There are three graphical methods which can be used to measure the SAF. Two of the methods use the amplitude part of a 1X-filtered Bode plot, and one method uses a 1X-filtered polar plot (Figure 1) [3].

The Half-Power Bandwidth method is endorsed by the API. The term "Half-Power" originated in AC electrical circuit theory, where the -3dB point corresponds to a voltage oscillation amplitude of 0.707 times the amplitude at resonance. Since simple electrical and mechanical oscillators are mathematically equivalent, the method can be applied to rotor systems; thus, the half-power points of the Bode plot have a vibration amplitude of about 70% of the maximum balance resonance amplitude. Once these two points have been identified, they are used to find the corresponding speed bandwidth. The difference between the half-power rotor speeds on each side of the resonance (Ω_{high} and Ω_{low}) defines the half-power bandwidth. These two rotor speeds are used together with the speed of maximum amplitude (the resonance speed Ω_{res}) to calculate the SAF:

$$SAF = \frac{\Omega_{res}}{\left(\Omega_{high} - \Omega_{low}\right)} \qquad (1)$$

The Peak Ratio method also uses the amplitude part of a 1X-filtered Bode plot. For an ideal rotor system, the ratio of the peak vibration amplitude at the balance resonance to the vibration amplitude at infinite running speed is equal to the SAF. In practice, the SAF can be measured by taking the peak vibration amplitude (A_{peak}) and dividing it by the vibration amplitude at a speed well above resonance (A_{fast}):

$$SAF = \frac{A_{peak}}{A_{fast}} \qquad (2)$$

Finally, a third method can be used, based on a 1X-filtered polar plot. For an ideal rotor system, the half-power points have a phase of 45° ahead of and 45° behind the phase at resonance. To use this method, a line is drawn from the origin of a compensated polar plot to the maximum amplitude of the resonance. Then, two additional lines are drawn from the origin at 45° left and 45° right of the first line. The rotor speeds associated with these three points are used in the same way as for the half-power bandwidth method:

$$SAF = \frac{\Omega_{res}}{\left(\Omega_{+45^\circ} - \Omega_{-45^\circ}\right)} \qquad (3)$$

One good thing about these three methods is that the graphical measure-

ments are independent of each other. Thus, they can be used as a check against mistakes in the calculations. Another thing to remember is that, because all real rotor systems deviate to some extent from the ideal, all three of these methods are approximate.

Several things can influence the accuracy of these measurements: uncompensated data, closely spaced shaft and casing modes, and split resonances. All plots must be compensated for slow roll runout before attempting the calculations. Closely spaced modes especially affect the Peak Ratio method, where the "high speed" response may be covered up by a higher mode or a mode from an adjacent machine. Split resonances can make the measurement of the SAF both difficult and dependent on angular orientation of the transducer used to measure the vibration.

Split resonances and the Synchronous Amplification Factor

A split resonance consists of two close resonances with a similar mode shape which are separated in speed. This is in contrast to two resonances which are independent modes and may have significantly different mode shapes.

Split resonances occur when a rotor system has anisotropic (asymmetric) stiffness; in other words, the rotor system static direct stiffness is not the same in all radial directions.

The rotor system direct stiffness consists of a combination of a large number of components which are chained together. It includes everything between "ground" and the rotor mass: foundation stiffness, bearing pedestal stiffness, the stiffness associated with any externally attached piping system, fluid bearing oil wedge stiffness, seal stiffness, and any rotor stiffness.

All of these components can have stiffness characteristics that are different in different radial directions. For example, bearing support pedestals on horizontal machines are often weaker horizontally than vertically. External

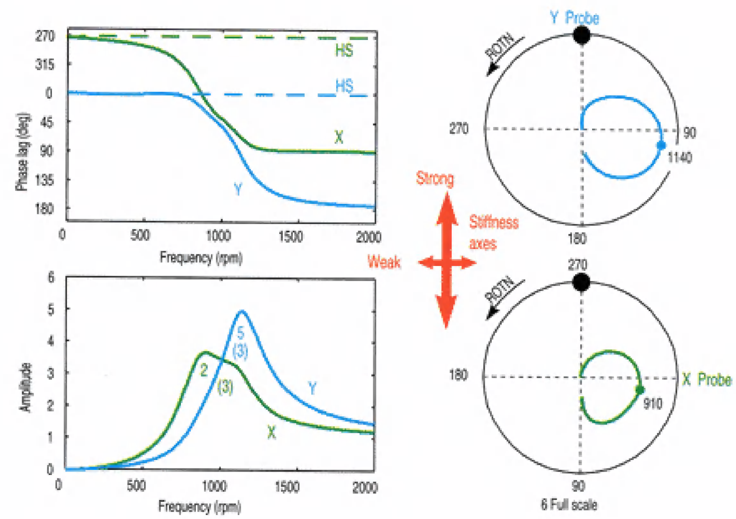

Figure 2
1X-filtered Bode and polar plots of a rotor system modeled with anisotropic stiffness. The measured responses from both coplanar probes are shown. The stiffness is weak horizontally and strong vertically. The probes are aligned with the stiffness axes. Note the difference in appearance of the two polar plots and that the split resonance peaks occur at different speeds. The numbers under the resonance peaks indicate the observed SAF using the Half-Power Bandwidth method and the Peak Ratio method (parentheses).

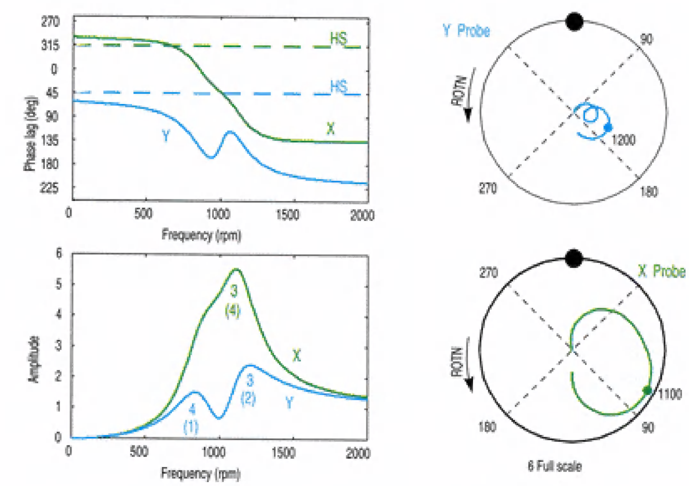

Figure 3
1X-filtered Bode and polar plots from the same model as in Figure 2. The probes have been rotated +45° (X to Y) relative to the horizontal and vertical stiffness axes. Note the internal loop in the Y-probe data which suggests a structural resonance. This loop is purely a result of the split resonance and the chosen viewpoint. Also note the large difference in the appearance of the resonance peaks. The numbers under the peaks indicate the observed SAF using the Half-Power Bandwidth method and the Peak Ratio method (parentheses). Note that the observed SAFs are different than those shown in Figure 2.

piping may be attached to a rigid casing horizontally, but not vertically, or vice versa. Fluid-lubricated bearings at a high eccentricity ratio can be significantly stiffer in the radial direction than in the tangential direction. The fluid bearing wedge thickness and orientation can be affected by the magnitude and direction of the static radial load. This load depends on process conditions and overall machine alignment, among other things. It is easy to see why almost all machines possess some degree of anisotropic stiffness.

It should be pointed out that it is the **Dynamic Stiffness** of the machine that controls the machine response at operating speeds. Besides the direct stiffnesses discussed above, the Dynamic Stiffness also includes mass, damping, and fluid circulation effects. Thus any mass, damping, or fluid circulation anisotropy will also contribute to the development of a split resonance. These effects, however, are usually smaller than the effects of anisotropic support stiffness.

The strong support stiffness can be assumed to be in a direction perpendicular to the weak stiffness. In general, the rotor system natural frequency, ω_n, is approximately,

$$\omega_n = \sqrt{\frac{K}{M}} \tag{4}$$

where K is the rotor system direct stiffness, and M is the rotor mass. Thus, a rotor system with anisotropic stiffness will have two natural frequencies associated with the different radial directions (for example, horizontal and vertical) of the strong and weak stiffness axes in the machine.

During a startup, a rotor system will pass through the resonance at the lower natural frequency first, only to encounter another resonance associated with the higher natural frequency. The width of the resulting split resonance will be determined by the difference between the two stiffnesses.

Typically, two orthogonal XY probes are installed in each observation plane in a machine. An *isotropic* rotor system (one which has the same direct stiffness in all radial directions) will produce identical X and Y probe polar plots (Figure 1) and circular orbits. However, a rotor system with *anisotropic* stiffness will produce polar plots which appear quite different from each other and orbits that are generally elliptical.

The degree of difference in these polar plots will depend on the degree of anisotropy. It will also depend on the angular orientation of the probes relative to the angular orientation of the anisotropic weak and strong stiffnesses in that plane. When probe axes are not aligned with stiffness axes, the difference between polar plots can become significant (Figures 2 and 3). Both sets of plots were generated using identical rotor models with horizontal (weak) and vertical (strong) stiffness axes. In Figure 2, the probes are *aligned with* the stiffness axes. In Figure 3, however, the probes are rotated 45° relative to the stiffness axes.

Note that, even when the probes are aligned with the stiffness axes (as in Figure 2), the polar plots differ in appearance. But, when the probes are rotated relative to the stiffness axes (as in Figure 3), the polar plots appear quite different. The Y probe polar plot has an internal loop which suggests a structural resonance. However, no such structural resonance exists in the model. This is purely a result of the anisotropic behavior of the rotor system. The Bode plots highlight the differences in phase response.

This sensitivity to probe orientation can lead to difficulty when trying to measure the SAF. The SAFs, measured using the Half-power Bandwidth and Peak Ratio methods, are shown as numbers beneath the resonance peaks in Figures 2 and 3. Note that the apparent SAFs of Figure 3 are quite different from each other and from the SAFs of Figure 2. Thus, if a measurement of the SAF were being made using the X probe data from Figure 3, one would arrive at a completely different conclusion than if one were using the Y probe data. This is why it is important to always generate Bode and polar plots, and SAFs, from *both* transducers in a plane!

So which Synchronous Amplification Factor is the "correct" one? Answer: none of the above. For machines with significant anisotropic stiffness, measurement of the SAF can become dependent on the measurement viewpoint and

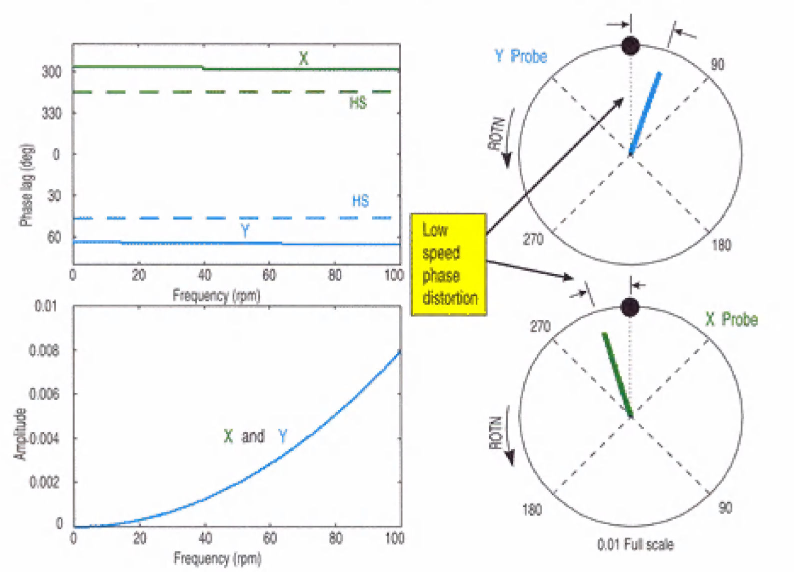

Figure 4
Low speed response of the model shown in Figure 3. The probes are rotated +45° from the orientation of the stiffness axes. The low speed phase lag angles do not point toward the heavy spot, making polar plot balancing more difficult.

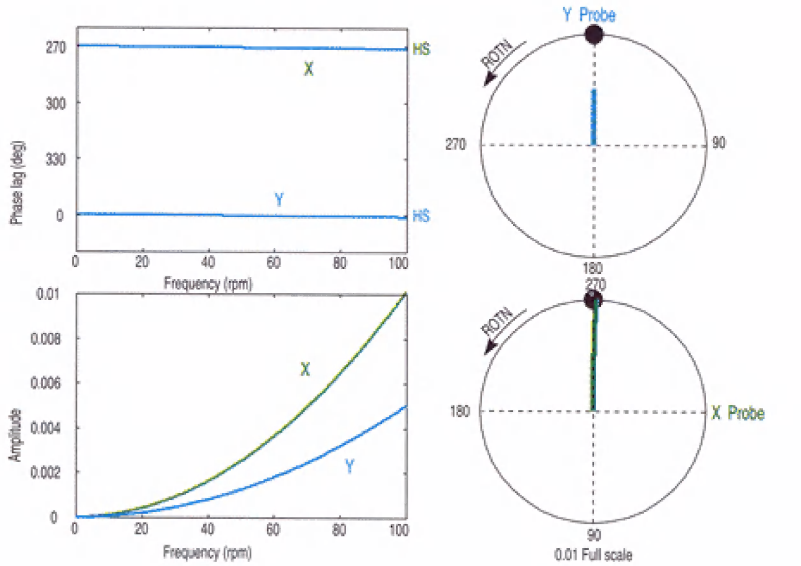

Figure 5

The low speed response of the anisotropic rotor model shown in Figure 2. The probes are now aligned with the stiffness axes, and the low speed phase lag angles point toward the heavy spot.

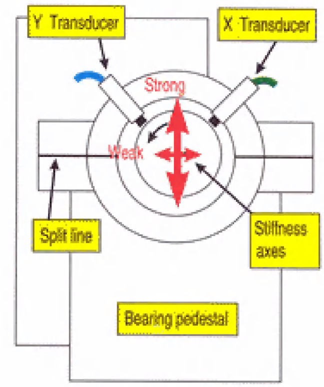

Figure 6

Schematic of a horizontally split machine. The tall bearing pedestal creates a rotor system support which is weaker horizontally than vertically. Probes are mounted ±45° from the vertical. This situation results in misalignment between the probe mounting orientation and the anisotropic stiffness axes, a situation modeled in Figure 3.

very difficult to interpret. This has important implications for machine acceptance testing.

Anisotropy can also lead to difficulty when trying to identify the location of the heavy spot for balancing. Figure 4 shows the low speed part of a polar plot from an anisotropic rotor model with the probes rotated 45° from the stiffness axes. The low speed phase does *not* point toward the heavy spot; both the X and Y probe plots have a phase distortion related to the degree of anisotropy and the probe orientation. Figure 5 shows the low speed behavior for the same model when the probes are aligned with the stiffness axes. In this case, the probes are mounted horizontally and vertically to coincide with the horizontal and vertical stiffness axis orientation. Note that the low speed phase from both probes now points towards the heavy spot. Analysis [1] has shown that, for synchronous behavior, ***when a pair of XY probes are aligned with the anisotropic stiffness axes, the low speed phase will be in the direction of the heavy spot.*** Reference [3] describes how the presence of a nearby fluid-lubricated bearing can also affect polar plot balancing.

Some machines, such as horizontally split compressors, are mounted on vertical bearing support pedestals. To avoid the split line in this type of machine, XY probe pairs are usually mounted with an angular orientation ±45° from the vertical (Figure 6). Because of the vertical bearing support pedestal structure and the gravity load of the rotor supported on fluid-lubricated bearings, the orientation of the stiffness axes may be close to horizontal and vertical (this assumes no other significant influences, such as process loads or misalignment). This is the same situation that was modeled in Figure 3. Because the probe axes are rotated relative to the rotor system stiffness axes, we would expect to see significant differences between the data from the X and Y probes in such a machine.

Virtual Probe Rotation

Given the sensitivity of vibration data to anisotropic stiffness and probe orientation, probes should be mounted so they are aligned with the stiffness axes. However, even if it were physically possible to do this, changes in the machine load, alignment, or support structure could cause the orientation of the stiffness axes to change.

Fortunately, the vector data from any XY probe pair can be mathematically transformed [1] to create a set of "virtual" probes at any desired angular orientation. Virtual Probe Rotation has been incorporated into the latest releases of Bently Nevada ADRE® and Data Manager® 2000 Software. Virtual Probe Rotation allows the user to examine the rotor response from many different probe mounting orientations to get a clearer picture of what is happening as the machine passes through resonances. Because measurement of the SAF can be very dependent on probe orientation, the user can select an orientation which provides the most appropriate viewpoint for calculating the SAF.

Virtual Probe Rotation can also be very helpful in identifying the location of the heavy spot for balancing machines with anisotropic stiffness. Probes can be mathematically rotated until the low

> *"Virtual Probe Rotation, now available in Bently Nevada ADRE® and Data Manager® 2000 Software, offers a new, powerful analysis tool for the machinery manager and diagnostician."*

speed response vectors of the compensated X and Y polar plots point in the same direction. That direction will be the direction of the heavy spot. When the probes are aligned with the stiffness axes, the compensated low speed phases of the probe pair on compensated Bode plots will differ by approximately 90°. (When the probes are *misaligned* with stiffness axes, the low speed phase can differ by considerably more or less than 90°.)

It must be emphasized that Virtual Probe Rotation requires vector data from two orthogonal, coplanar probes. Also, the two transducers must be of the same type (such as displacement). Data from single probes cannot be transformed in this manner.

This brings up a good point. The figures presented here show that a one-probe-per-plane machine monitoring system, while better than nothing, is quite limited in its ability to show what is happening in rotor systems with a significant degree of anisotropic stiffness. As we have seen, the polar and Bode plots from two orthogonal probes can appear quite different from each other. A diagnostician who used only the Y probe data from Figure 3 would have a very misleading picture of what was actually happening in the machine.

Summary

There are three graphical methods for estimating the Synchronous Amplification Factor of a machine. The methods are independent, but all three are derived from ideal rotor models. Because real rotor systems deviate from ideal behavior, all of these methods are approximate.

Anisotropic rotor system stiffness tends to produce elliptical orbits and split resonances. When split resonances are present, Bode and polar plots from X and Y probes in the same plane can appear very different. Also, the appearance of Bode and polar plots will be different, depending on the relative angular orientation of the probes and stiffness axes. The combination of these effects can make determination of the SAF difficult and viewpoint dependent. This has important implications for machine acceptance testing.

Rotor system support anisotropy, because it affects measurement of low speed phase, can also make polar plot balancing more difficult. In anisotropic rotor systems, the low speed response vectors of X and Y polar plots may not agree with each other as to the location of the heavy spot.

Vector data from a coplanar XY probe pair can be mathematically transformed to create a set of rotated, "virtual" probes at any desired angular orientation. Virtual Probe Rotation allows us to "view" the rotor from several different aspects and observe changes in the observed SAF.

Virtual Probe Rotation is also useful for polar plot balancing of anisotropic machines. By rotating the virtual probes to align them with the stiffness axes, the low speed phases of a pair of XY probes will agree as to the location of the heavy spot. Thus, it becomes easier to make a decision on initial weight placement in the balancing process.

Virtual Probe Rotation, now available in Bently Nevada ADRE and Data Manager 2000 Software, offers a new, powerful analysis tool for the machinery manager and diagnostician.

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